TELLUSANT

Streamlining Corporate Decision Making

Strategic Pricing

Technical Approach

2024

Theoretical background I: Conditions that always shall be met

By meeting these conditions, runaway outcomes are avoided

Marshall's Homogeneity Condition

$$E_{i1} + E_{i2} + \cdots + E_{ii} + \cdots + E_{in} + E_{iI} = 0$$

where

E_{ij} = price elasticity of demand for good i with price change of good j

E_{iI} = income elasticity of demand for good i

Without this, it is possible to create something out of nothing.

Hotelling-Jureen Symmetry Condition

$$E_{ij} = \frac{S_i}{S_j} E_{ji} + S_j(E_{jI} - E_{iI})$$

where

S_i = value share of good i

Without this, cross-elasticities give different results when looking at the market from the point of view of category i versus category j.

Engel Condition

$$s_1 E_{1I} + s_2 E_{2I} + \cdots + E_{in} = 1$$

where

S_i = value share of good i

Of no importance here except it should not be violated if elasticities are included in a total category analysis

Elasticity is usually denoted by epsilon ϵ. E is here capital epsilon but can be called e

Basic elasticity equation

This is the equation most companies use, with a constant elasticity

$$\epsilon = \frac{\Delta y}{y} / \frac{\Delta x}{x} \rightarrow$$

$$\epsilon = \lim_{\Delta \to 0} \frac{x\Delta y}{y\Delta x} = \frac{\mathrm{x}}{\mathrm{y}} \frac{dy}{dx}$$

Elasticity ϵ is the change in outcome (e.g., volume) from a change in a condition (e.g., price)

Integrate

$$lny = \epsilon lnx + C$$

By integrating we a) get a linear regression specification, and b) see why we transform with the logarithm

$$y = Cx^{\varepsilon}$$

We get untransformed power curve by taking the exponent of both each side

"…[the linear regression model]…is not consistent with utility maximization…and will eventually lead to gross over-prediction"
Angus Deaton, Nobel Prize winner for research on consumption patterns

Core elasticity equation

For all the demand drivers we look at, we always include the **core elasticity equation** (often called a short regression). Without these, other drivers like spirits cross-elastic, impact of inflation or importance of mega players will be mis-estimated

$$y'(x) - \eta \frac{y(x)}{x} + \zeta y(x) = 0$$

This differential equation introduces a damping factor ζ to the linear equation. This is now a nonlinear equation

Core equation with damping

$$y(x) = c_1 \cdot x^{\eta} \cdot e^{-\zeta x}$$

Base elasticity eta

Damping factor zeta

Core equation with damping

We simplify terminology

$$\mathrm{H}\ (capital\ eta) = \eta$$
$$\mathrm{Z}\ (capital\ eta) = \zeta$$
$$vol\ = y$$
$$di(disposable\ income\ per\ capita) = x_1$$
$$p\ (price) = x_1$$

For simplicity
aitch
zed or zet

Simplistic equations without true interaction effect between price and income

(1) Price, linear eq. $vol_B = p_B^{\beta_0} \cdot p_1^{\beta_1} \cdot p_2^{\beta_2} \cdot p_3^{\beta_3} \cdot di^{\beta_4} \cdot e^{\varepsilon}$

(2) Price, quadratic eq. $vol_B = p_B^{\beta_0} \cdot p_1^{\beta_{1L}} \cdot (p_1^2)^{\beta_{1Q}} \cdot p_2^{\beta_{2L}} \cdot (p_2^2)^{\beta_{2Q}} \cdot p_3^{\beta_{3L}} \cdot (p_3^2)^{\beta_{3Q}} \cdot di^{\beta_4} \cdot e^{\varepsilon}$

(3) Price index, linear eq. step 1 $vol_B = p_B^{\beta_0} \cdot pi_1^{\beta_{i1}} \cdot pi_2^{\beta_{i2}} \cdot pi_3^{\beta i_3} \cdot di^{\beta_4} \cdot e^{\varepsilon}$

(4) Price, linear eq. step 2 $vol_B = p_B^{\beta_0} \cdot \left(\frac{p_1}{p_B}\right)^{\beta_{i1}} \cdot \left(\frac{p_2}{p_B}\right)^{\beta_{i2}} \cdot \left(\frac{p_3}{p_B}\right)^{\beta_{i3}} \cdot di^{\beta_{i4}} \cdot e^{\varepsilon}$

(5) Price, linear eq. step 3 $vol_B = p_B^{(\beta_0 - \beta_{i1} - \beta_{i2} - \beta_{i3})} \cdot p_1^{\beta_{i1}} \cdot p_2^{\beta_{i2}} \cdot p_3^{\beta i_3} \cdot di^{\beta_{i4}} \cdot e^{\varepsilon}$

(6) Price index, quadratic eq. is trivial: follows from (2) and (4). Probably not needed

Core elasticity equation for income <u>and</u> price

These equations are s-curves

$$vol(di,p) = c_1(di^{H_1} \cdot e^{-D_1 di})(p^{H_2} \cdot e^{-D_2 p})$$

This equations give elasticities for income and price independent of each other

$$E_{di} = H_1 - D \cdot di$$
$$E_p = H_2 - D \cdot p$$

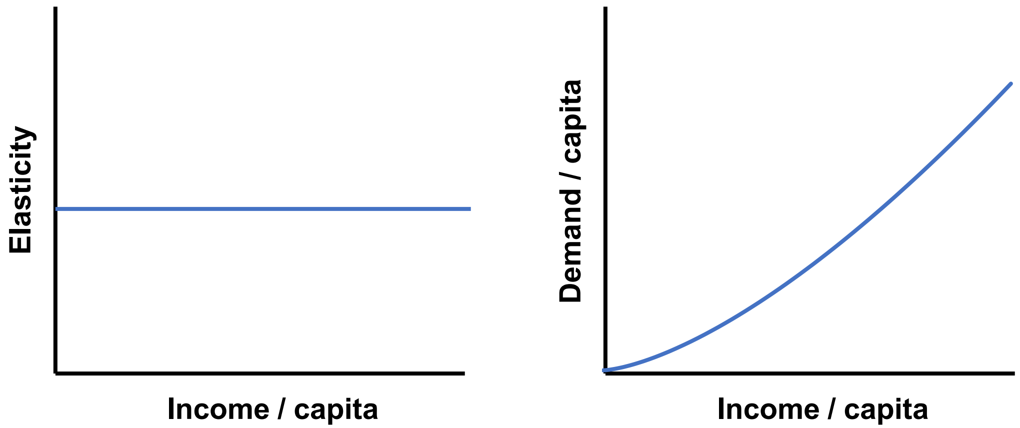
From this follows the income and price elasticities (without proof).

$$vol(di,p) = c_1(di^{H_1} \cdot e^{-D_1 di})(p^{H_2} \cdot e^{-D_2 p})(di \cdot p)^{H_3}$$

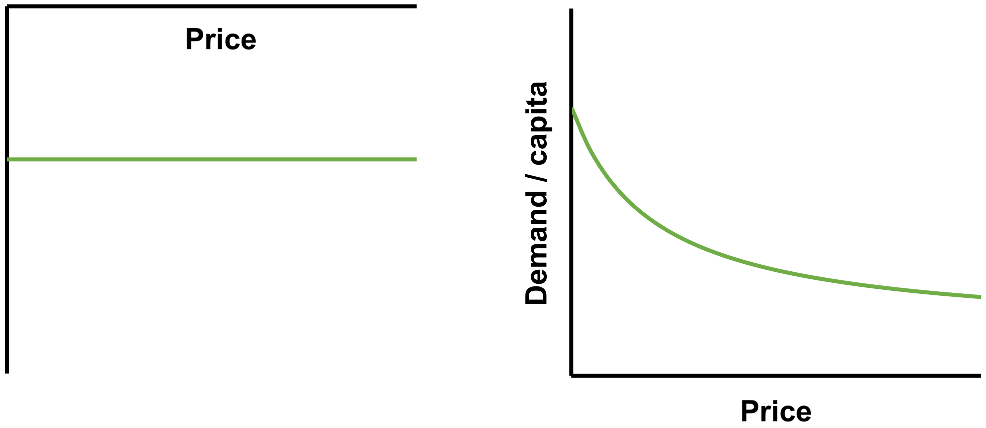
We add an interaction effect so that income and price elasticities are a function of income and price simultaneously

Visually, the equations are nonlinear

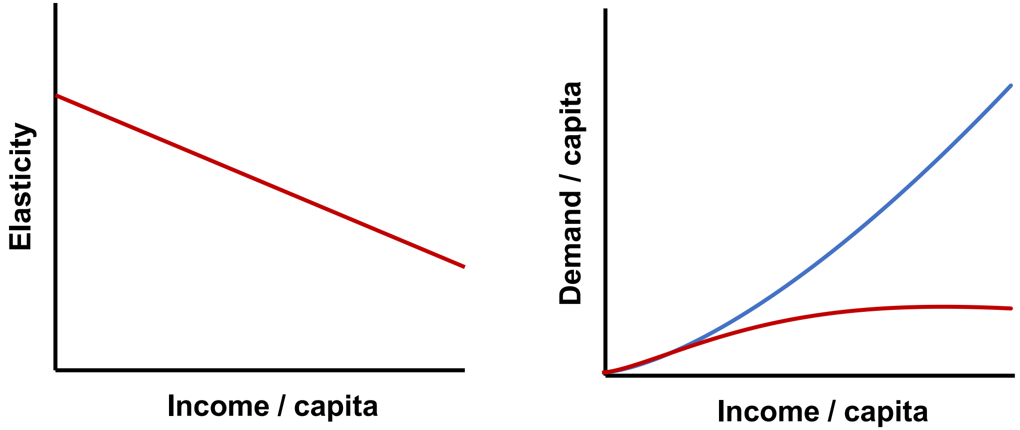
UNDAMPED INCOME EFFECTS



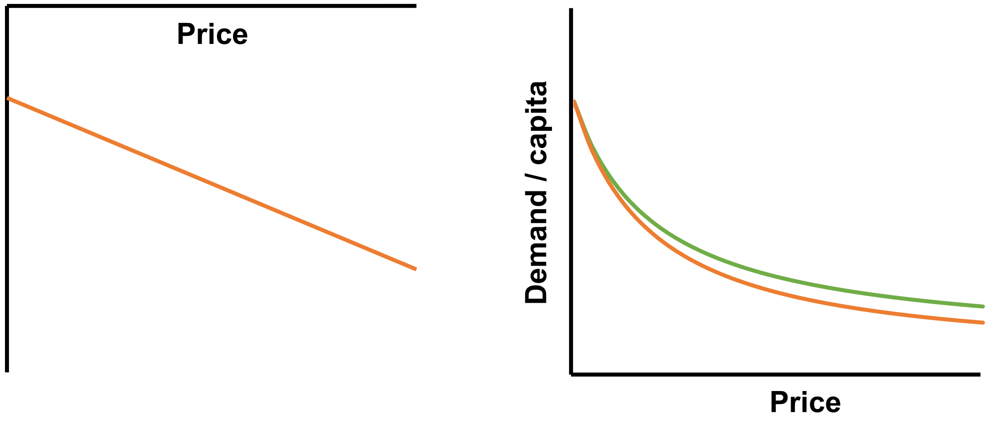
UNDAMPED PRICE EFFECTS



DAMPED INCOME EFFECTS



DAMPED PRICE EFFECTS



Assumptions for modelling: ergodicity

Systems are ergodic when spatial (cross-sectional) snapshots mirror historical evolutions (timeseries)

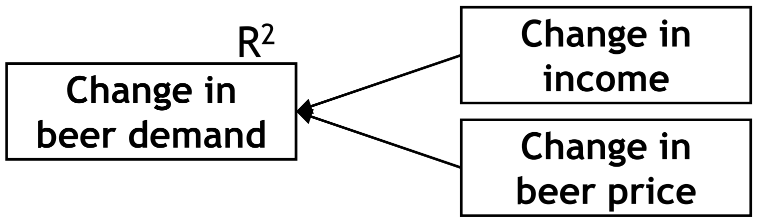
"In practice, the choice between using cross-sectional or time series analysis largely depends on the nature of the data"

“If a variable exhibits consistent patterns across different entities at a specific point in time, and these patterns are also stable over time, then cross-sectional and time series analyses give identical results.”

Path diagrams are used to describe models

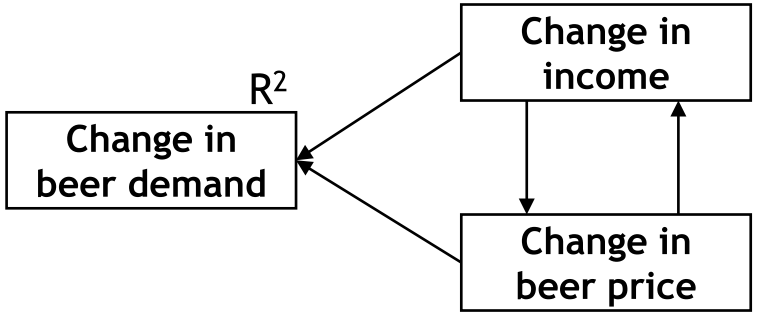
SIMPLIFIED PATH DIAGRAM

$$vol(di,p) = c_1(di^{H_1} \cdot e^{-D_1 di})(p^{H_2} \cdot e^{-D_2 p})$$



Coeff^sig for arrow

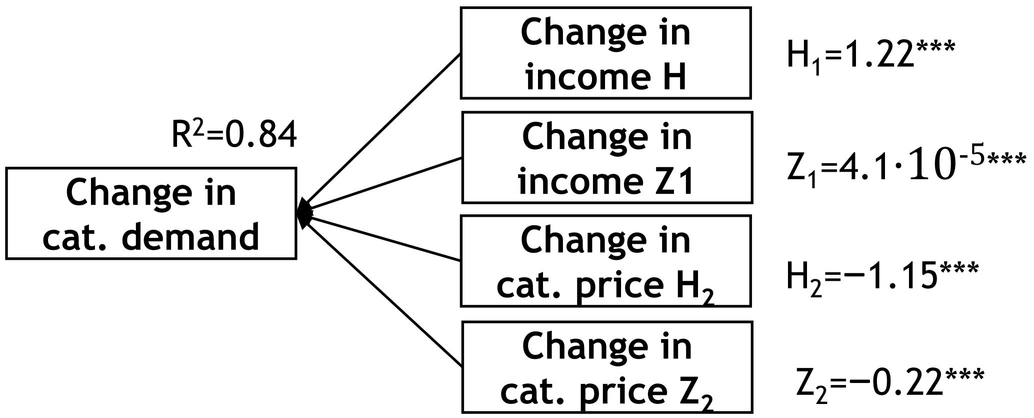
$$vol(di,p) = c_1(di^{H_1} \cdot e^{-D_1 di})(p^{H_2} \cdot e^{-D_2 p})(di \cdot p)^{H_3}$$



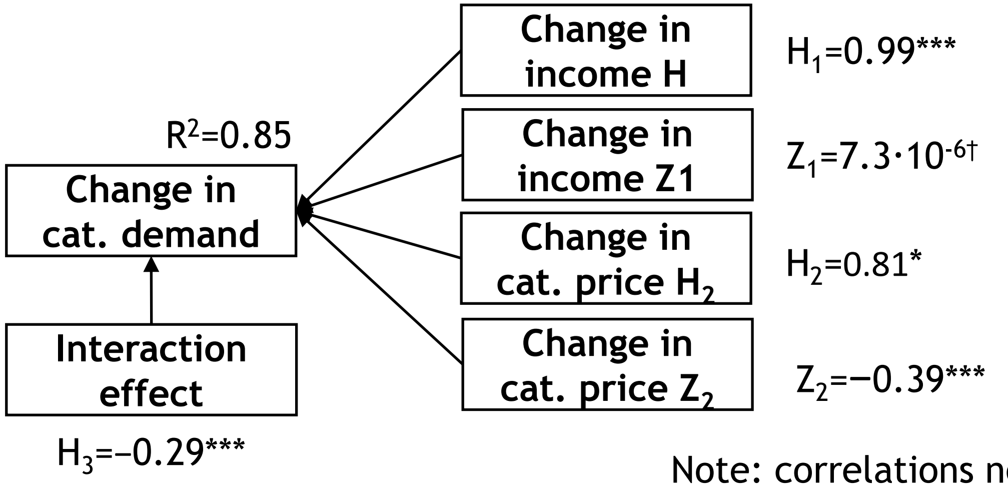
Note: correlations not shown for simplicity
Sig(nificance) expressed as † p<10%, * p<5%, ** p<1%, *** p<0.1% (two-tailed)

Short model without interaction

CORE MODEL W/O INTERACTION



CORE MODEL W/ INTERACTION

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. nl ( pcc={c_1}*exp(-{zeta1}*di_cap)*di_cap^{eta1}*exp(-{zeta2}*p)*p^{eta2}) if
(obs = 1,756)


      Source |      SS            df       MS
-------------+----------------------------------    Number of obs =      1,756
       Model |  4024134.2          5  804826.838    R-squared     =     0.8409
    Residual |  761348.92       1751  434.808062    Adj R-squared =     0.8405
-------------+----------------------------------    Root MSE      =   20.85205
       Total |  4785483.1       1756  2725.21817    Res. dev.     =   15645.84

------------------------------------------------------------------------------
         pcc |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        /c_1 |   .0013795   .0007513     1.84   0.066    -.0000939     .002853
      /zeta1 |   .0000414   3.94e-06    10.50   0.000     .0000337    .0000492
       /eta1 |   1.222315   .0630578    19.38   0.000     1.098639    1.345992
      /zeta2 |  -.2242883   .0185202   -12.11   0.000    -.2606124   -.1879642
       /eta2 |  -1.151031   .0661272   -17.41   0.000    -1.280728   -1.021335
------------------------------------------------------------------------------
```

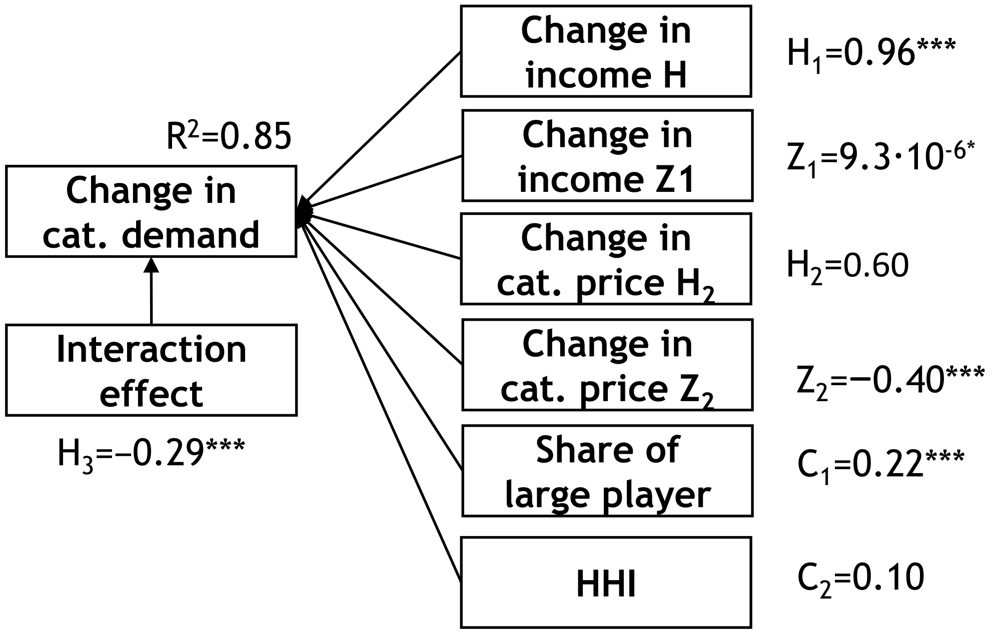
The added interaction effect make numbers hard to compare with ‘w/o interaction’, without multiplying out the formulas

Note: correlations not shown for simplicity
Significance expressed as † p<10%, * p<5%, ** p<1%, *** p<0.1% (two-tailed)

With short model as base, augmented long models are built (also called long regressions)

- Cross-elasticity with carbonates
- Cross-elasticity with spirits
- Cross-elasticity with wine
- Impact of inflation
- Impact of having a mega player
- Impact of market concentration (HHI)
- Other

EXAMPLE: IMPACT OF MEGA PLAYER AND MARKET CONCENTRATION (HHI*)



* HHI is Hirschman-Herfindahl index, a measure of market concentration

This document only shows technical methods.